Technical Report No. 32-201

Vortex-Tube and Regenerative-Cooling-Tube Parameters for Gaseous Fission Reactors

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ABSTRACT

The performance of the vortex-tube reactor is governed primarily by the propellant mass flow rate, radius, and number of vortex tubes. A simple analysis is carried out to define roughly the range of variables for which system performance is attractive. It is shown that the ratio of the radiation terms ε_c/β establishes the allowable range of values for the vortex-tube parameters. Whether or not the required values of ε_c/β can be obtained while maintaining adequate system capabilities depends upon the solution of the thermal radiation problem. The regenerative-cooling-tube parameters depend primarily upon the cooling-tube void fraction.

I. INTRODUCTION

The performance potential of nuclear rockets has to date been limited by temperature constraints imposed by the structural requirements of the fuel-bearing region of the reactor. The conventional methods of power removal result in rather modest operating temperatures and enthalpy gains by the propellant; hence, the full potential of the fission process cannot be realized. One method of uncoupling the nuclear system from these constraints is to employ a fission reactor with a non-temperature-limited or gaseous region of fuel-bearing material. For these systems, the major problem is providing an efficient means of separating the fissionable species from the propellant before the latter is expelled. One particular concept of the gaseous fuel reactor employs vortex containment, which allows intimate mixing of the propellant and fissionable material. The fuel-bearing medium exists in two different forms: a temperature-limited or solid region which produces a fraction f of the total power and a gaseous region where the remainder of the power is developed and for which no maximum allowable temperature is specified.

Figure 1 shows a conceptual arrangement of the system. The temperature-limited central region may consist of solid fuel elements or a circulating fluid fuel, and the surrounding annular region is considered to be made up

of a system of cylindrical cavities, each of which contains a vortex flow field. The propellant regeneratively cools the central region and is heated from storage conditions to temperature T_s. It then flows through the vortex tubes, where it is further heated by intimate mixing with the fuel and is expelled at temperature T_c. A radiator may be incorporated into the system to dispose of all or part of the heat generated in the solid region; this effectively removes the upper bound on the specific impulse. Systems with radiators will not be considered in this Report.

Previous analyses (Ref. 1) of gaseous fission reactors have been carried out for both low- and high-acceleration missions. Since the performance of the system is governed to a large extent by the mass flow rate, radius, and number of vortex tubes, it is of interest to determine the value of these parameters for different missions. For analytical reasons, very simple and highly idealized reactor and engine models are utilized, and as such, the results will be optimistic and will define roughly the range of variables for which the system performance is attractive.

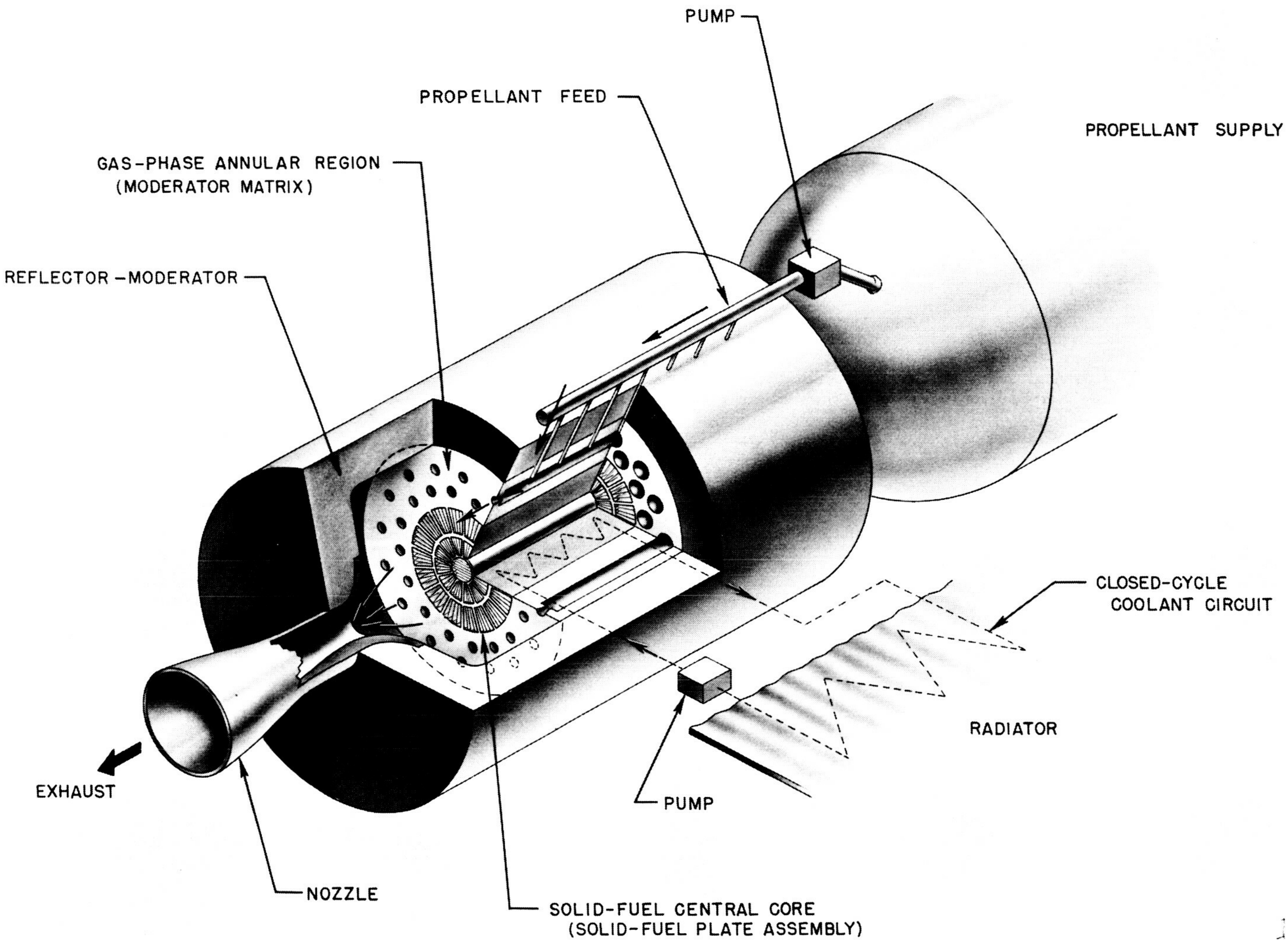

Fig. 1. Conceptual arrangement of nuclear rocket engine with non-temperature-limited fuel-bearing region and radiator system

II. METHOD OF CALCULATION

A. Assumptions

1. The performance calculations are based upon simple "burnout velocities" of single-staged vehicles in vertical, drag-free flight in a constant gravitational field.
2. The gross weight of the vehicle consists of the payload, propellant, tanks, and reactor.
3. The solid regions of the reactor are regeneratively cooled by the propellant flowing through circular tubes uniformly distributed throughout the solid.
4. The vortex flow is two-dimensional, laminar, and inviscid.
5. The propellant regeneratively cools the nozzle and enters the reactor in the gaseous phase at 900°R.

B. Equations of Interest

The performance potential of the system may be obtained in terms of the specific impulse I_c corresponding to the enthalpy h_c. If the propellant at stagnation enthalpy h_c is completely expanded in a de Laval nozzle, then

$$I_c = \frac{1}{g}\sqrt{2h_c} \tag{1}$$

A power balance for the system (Ref. 2) then yields

$$I^{*3} + \frac{f + \zeta(1 - f)}{\beta} I^{*2} - \frac{\beta + 1}{\beta} = 0 \tag{2}$$

where

$$\beta \equiv \frac{\sigma \varepsilon_c A_c T_s^4}{\dot{m} h_s}$$

The gross vehicle weight is considered to be made up of the payload, propellant, tanks, and reactor, and it is easy to show that the payload ratio is

$$\pi_{PL} = 1 - \lambda(1 + s) - \frac{a}{\theta I^*} \tag{3}$$

where λ satisfies

$$\ln(1 - \lambda) + \frac{\lambda}{a} = -\frac{V}{I^*} \tag{4}$$

The second term of Eq. (3) is the contribution of the propellant and tanks, and the third term is that of the reactor. Since the ratio of the power released in the solid to the total power generated is

$$\frac{P_s}{P} = \frac{\dot{m} h_s}{\dot{m} h_c} = \frac{1}{I^{*2}} \tag{5}$$

it can be shown that

$$P = \frac{F c_s I^*}{2} \tag{6}$$

where

$$F = a W_0 = a \frac{W_{PL}}{\pi_{PL}} \tag{7}$$

and

$$c_s = g I_s = \sqrt{2h_s} \tag{8}$$

If the allowable power density in the solid p_s is fixed, the reactor volume is

$$V_R^* = \frac{P}{p_s I^{*2}(1 - \chi - \kappa)} \tag{9}$$

and for a cylindrical reactor of height equal to diameter,

$$l = \left(\frac{4}{\pi} V_R^*\right)^{1/3} \tag{10}$$

By definition,

$$\beta = \frac{\sigma \varepsilon_c A_c T_s^4}{\dot{m} h_s} \tag{11}$$

and the required vortex-tube surface area is therefore

$$A_c = \frac{1}{\sigma T_s^4}\left(\frac{\beta}{\varepsilon_c}\right)\left(\frac{P}{I^{*2}}\right) \tag{12}$$

If the vortex-tube cavity fraction χ is fixed, then

$$\chi V_R^* = \pi r_V^2 l N_V \tag{13}$$

and

$$A_c = 2\pi r_V l N_V \tag{14}$$

Solving Eqs. (13) and (14) for r_V yields

$$r_V = \frac{2\chi V_R^*}{A_c} \tag{15}$$

From Eq. (13),

$$N_V = \frac{A_c}{2\pi l r_V} \tag{16}$$

An analysis of two-dimensional, laminar, inviscid vortex flow (Ref. 3) has shown the mass flow rate per unit length of tube to be independent of tube radius; thus,

$$\dot{m} = \mathfrak{M} l N_V \tag{17}$$

Combining Eqs. (5), (8), and (17) yields

$$\mathfrak{M} = \frac{2}{c_s^2} \frac{P}{l N_V I^{*2}} \tag{18}$$

If the regenerative-cooling-tube void fraction κ is chosen, then the following relations hold:

$$\kappa V_R^* = \pi r_r^2 l N_r \tag{19}$$

$$A_r = 2\pi r_r l N_r \tag{20}$$

and

$$P_s = \dot{m} h_s = h^* A_r \Delta T_r \tag{21}$$

Using the Colburn relation for the heat-transfer coefficient,

$$h^* = 0.023 \frac{k}{2r_r} \left(\frac{2V\rho r_r}{\mu} \right)^{0.8} (Pr)^{0.4} \tag{22}$$

and, combining with Eqs. (19), (20), and (21), the regenerative-cooling-tube radius is

$$r_r = \left[\frac{0.023 k \kappa \Delta T_r}{p_s (1 - \chi - \kappa)} \right]^{5/6} \left(\frac{8 \mathfrak{M} N_V}{\pi \mu l \kappa} \right)^{2/3} (Pr)^{1/3} \tag{23}$$

From Eq. (20), the required number of regenerative-cooling tubes is

$$N_r = \frac{\kappa V_R^*}{\pi r_r^2 l} \tag{24}$$

Relations (15), (16), (18), (23), and (24) have been solved for a series of values of α, β, ε_c, χ, and κ in the case of an Earth-satellite mission ($V = 1.14$) and an Earth-escape mission ($V = 1.65$). For all of these calculations, $a = 1.3$, $\zeta = 0.1$, $W_{PL} = 10^5$ lb, $s = 0.05$, $\theta = 20$, $p_s = 2.68 \times 10^4$ Btu/ft^3-sec (1 kw/cm^3), $c_s = 22{,}400$ ft/sec, $\rho_s = 92.35$ lb/ft^3 (1.48 g/cm^3), $T_s = 3600$°R (2000°K) and $I_s = 700$ sec.

III. DISCUSSION

A. Vortex-Tube Mass Flow Rate $\mathfrak{M}$

The expression for the vortex-tube mass flow rate (Eq. 17) may be put in terms of the variables ε_c, β, χ, and κ, so that

$$\mathfrak{M} \sim \left(\frac{\varepsilon_c}{\beta}\right)^2 \frac{\chi}{1-\chi-\kappa} \tag{25}$$

It is clear from Eq. (25) that $\mathfrak{M}$ depends only upon the ratio of the radiation parameters ε_c/β and the cavity-to-solid volume-fraction ratio. Fluid-dynamic considerations require that $\mathfrak{M} \sim 0.01$ to 0.02 lb/ft-sec if an adequate separation process is to result. From Fig. 2, it can be seen that this implies $\varepsilon_c/\beta \sim 20$ for void fractions of ~ 0.1 and $\varepsilon_c/\beta \sim 5$ for void fractions of ~ 0.6.

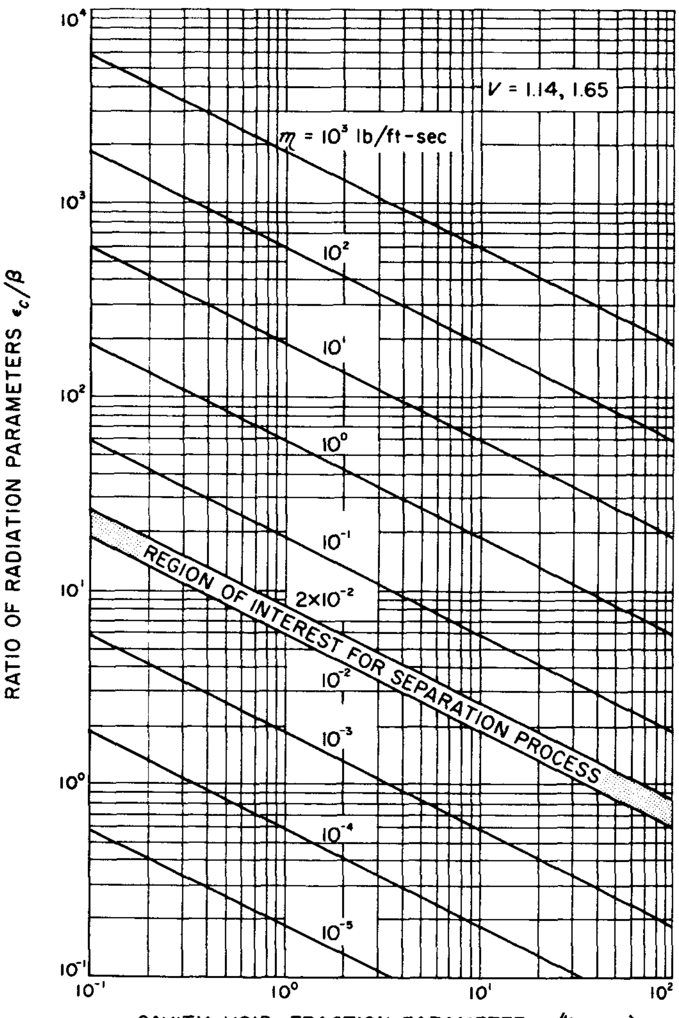

Fig. 2. Ratio of radiation parameters as a function of cavity-void-fraction parameter for various vortex-tube mass flow rates

The performance of the system is rapidly degraded as β increases; past analyses (see Ref. 1) have shown that for the high-acceleration systems, a β of 10^{-2} is a reasonable value. This, of course, implies that ε_c range from 0.05 to 0.20.

Whether these values of ε_c/β can be achieved without greatly degrading system capabilities is dependent upon the resolution of the thermal-radiation problem; that is, the determination of the numerical value of ε_c.

B. Vortex-Tube Radius r_V

If Eqs. (9), (12), and (15) are combined, the result is that

$$r_V \sim \frac{\varepsilon_c}{\beta} \frac{\chi}{1-\chi-\kappa} \tag{26}$$

As before, only the radiation parameters and void fractions are important. From Fig. 3, the vortex-tube radii corresponding to the required mass flow rates are in the range of 0.13–6.00 in.

C. Number of Vortex Tubes N_V

The number of vortex tubes can also be related to the radiation parameters and void fractions through the ratios ε_c/β and $\chi/(1-\chi-\kappa)$, but, in addition, the parameter P/Π^{*2} appears. That is,

$$N_V \sim \left(\frac{\beta}{\varepsilon_c}\right)^2 \frac{1-\chi-\kappa}{\chi} \frac{P}{\Pi^{*2}} \tag{27}$$

Since the quantity P/Π^{*2} depends upon the variables V, α, χ, κ, and β in a complex fashion, it is not expected that N_V can adequately be represented graphically with the two groups of variables ε_c/β and $\chi/(1-\chi-\kappa)$. An examination of Figs. 4 and 5 reveals that

1. N_V increases as α increases, the effect being more pronounced for small values of α.
2. N_V increases as β increases, the effect being more pronounced at large cavity fractions.
3. The variations due to χ, κ, and ε_c are adequately accounted for by the parameters ε_c/β and $\chi/(1-\chi-\kappa)$.

In order to obtain the proper mass flow rates, the required number of vortex tubes will be of the order of 3,000 to 20,000 when $V = 1.14$ and will increase to

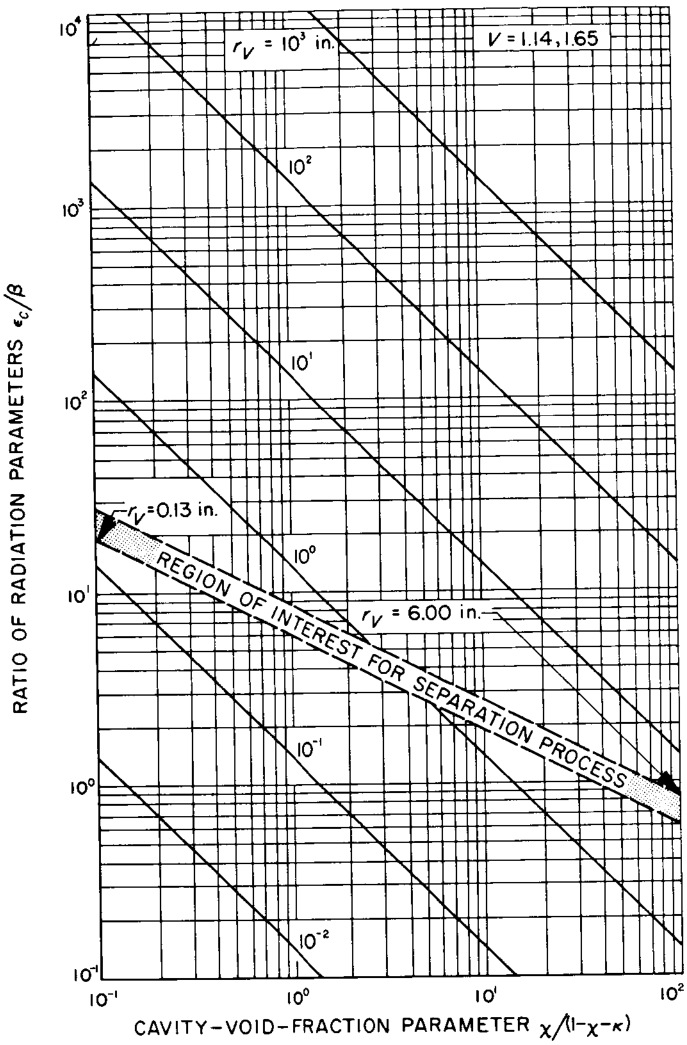

Fig. 3. Ratio of radiation parameters as a function of cavity-void-fraction parameter for various vortex-tube radii

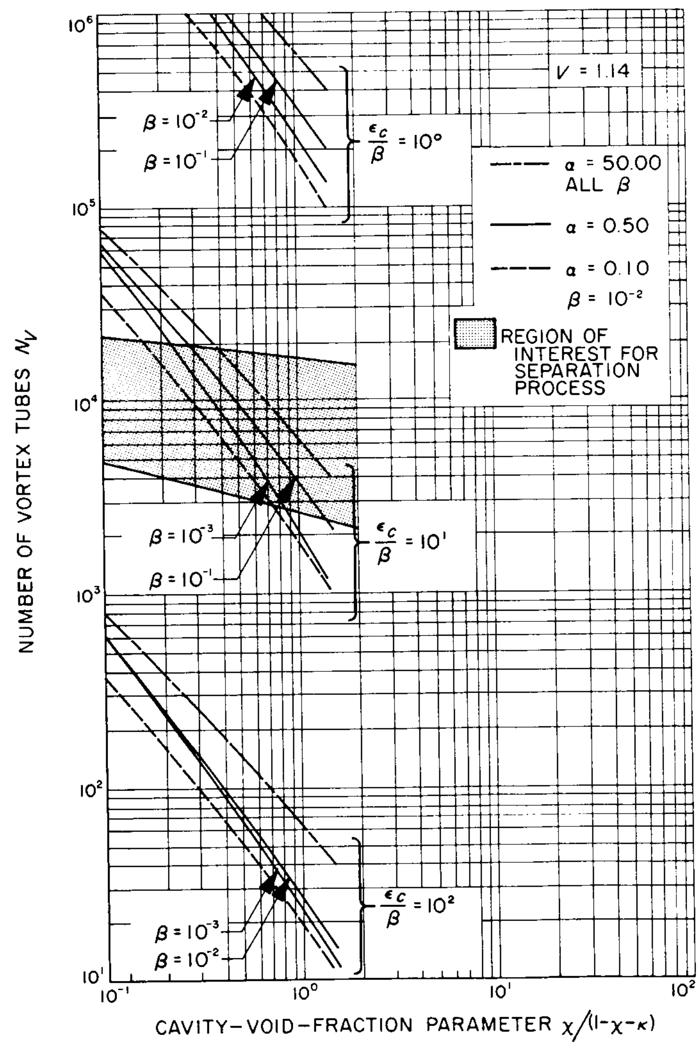

Fig. 4. Number of vortex tubes as a function of cavity-void-fraction parameter, $V = 1.14$

100,000 for the higher-performance Earth-escape mission ($V = 1.65$) and small void fractions. It is evident from the curves that small β ($< 10^{-3}$), large cavity fractions ($\chi > 0.6$), and large fractions of the fuel in the gas phase ($\alpha << 1$) are desirable if the number of vortex tubes is to be substantially decreased. Unfortunately, the last two requirements lead to large reactor volumes.

D. Regenerative-Cooling-Tube Radius r_r

By combining Eqs. (10), (16), (18), and (23), the regenerative-cooling-tube radius is found to be

$$r_r \sim \left(\frac{\kappa}{1-\chi-\kappa}\right)^{1/6} l^{2/3} \qquad (28)$$

Because of the factor $l^{2/3}$, r_r is not a simple function of the void fraction ratio $[\kappa/(1-\chi-\kappa)]$ but depends upon α, V, and β as well. From Figs. 6 and 7, the following observations can be made:

1. An increase in β increases r_r, since the radiative heat load in the solid increases, thus requiring more heat-transfer area, and hence, larger r_r. The effect is greatest for small α and large χ. For large values of α, a change in β has almost no effect on r_r, since the radiative heat load is then but a small fraction of the heat generated in the solid.
2. An increase in α increases r_r, because the fraction of the power generated in the solid is larger and greater heat-transfer area is required. The effect is most pronounced at small α and large χ.
3. A change in χ has a small effect on r_r.
4. An increase in V increases r_r, because power requirements increase. The effect is greater for larger α.
5. The variation due to κ is adequately accounted for by the parameter $\kappa/(1-\chi-\kappa)$.

For the range of the variables considered, r_r changes from 0.15 to 0.60 in.

E. Number of Regenerative-Cooling Tubes N_r

The required number of regenerative-cooling tubes can be shown to be

$$N_r \sim \left(\frac{1-\chi-\kappa}{\kappa}\right)^{1/3} \kappa l^{2/3} \qquad (29)$$

Again, the factor $l^{2/3}$ makes N_r dependent upon α, V, and β. From Figs. 8 and 9, the following is evident:

1. An increase in β increases N_r, since the heat load in the solid region is greater. The effect is most

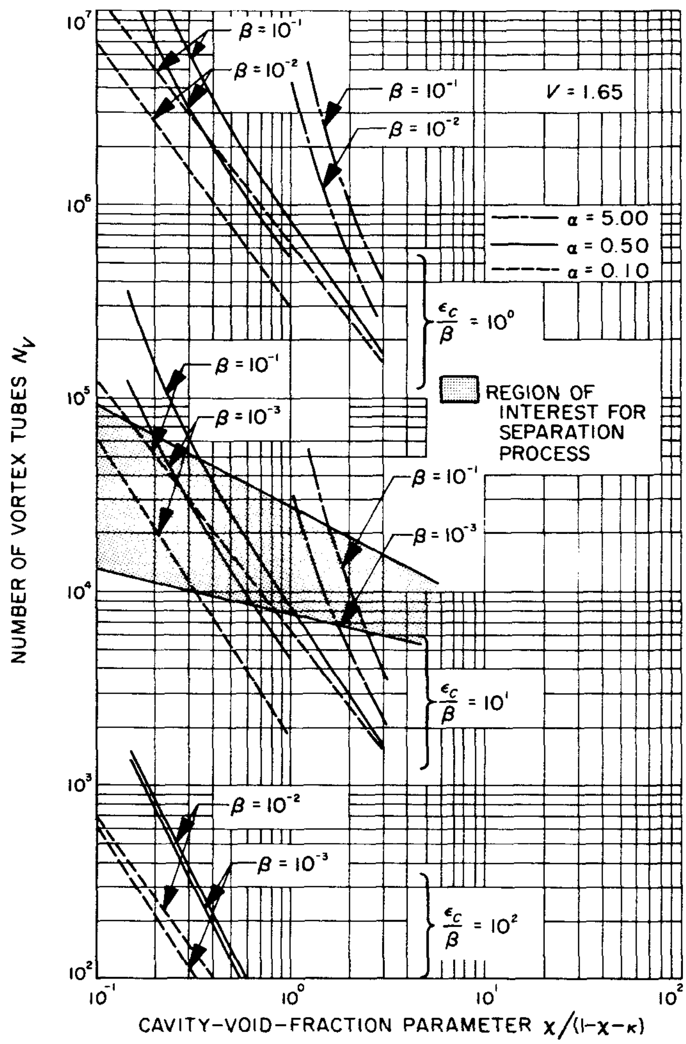

Fig. 5. Number of vortex tubes as a function of cavity-void-fraction parameter, $V = 1.65$

noticeable at small α; β has little effect on N_r at large values of α.

2. An increase in α increases N_r; the change is greatest for small χ and κ.
3. An increase in χ decreases N_r, the greatest effect occurring for small α and κ.
4. The number of regenerative tubes is most sensitive to κ. An increase in κ by a factor of 15 increases the number of tubes by a factor of 6. This results from the fact that for a constant heat load, large κ implies low gas velocities, small heat-transfer coefficients, and large surface area, and hence, more tubes.

In establishing the proper value of κ, the pressure drops must be considered.

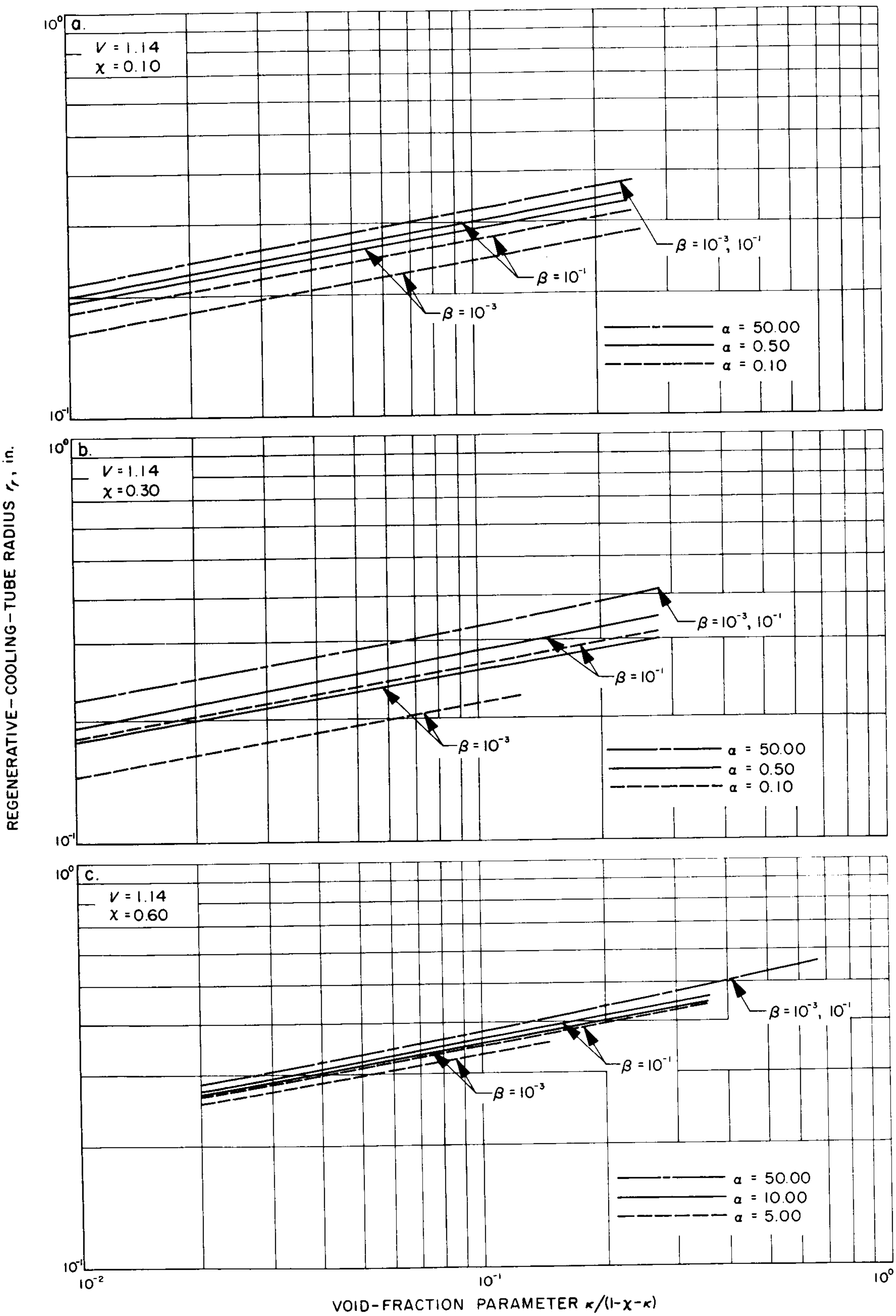

Fig. 6. Regenerative-cooling-tube radius as a function of void-fraction parameter, $V = 1.14$

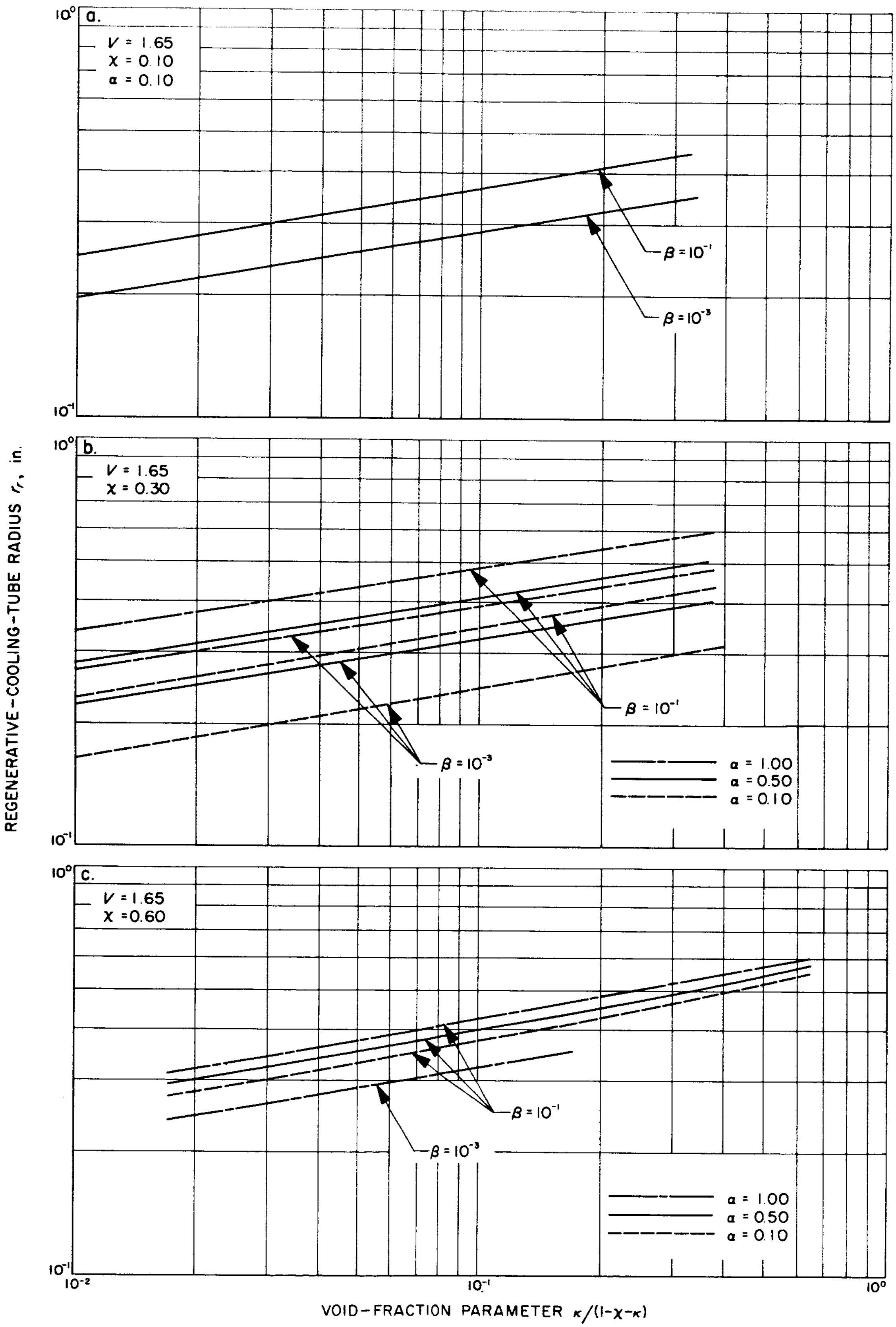

Fig. 7. Regenerative-cooling-tube radius as a function of void-fraction parameter, $V = 1.65$

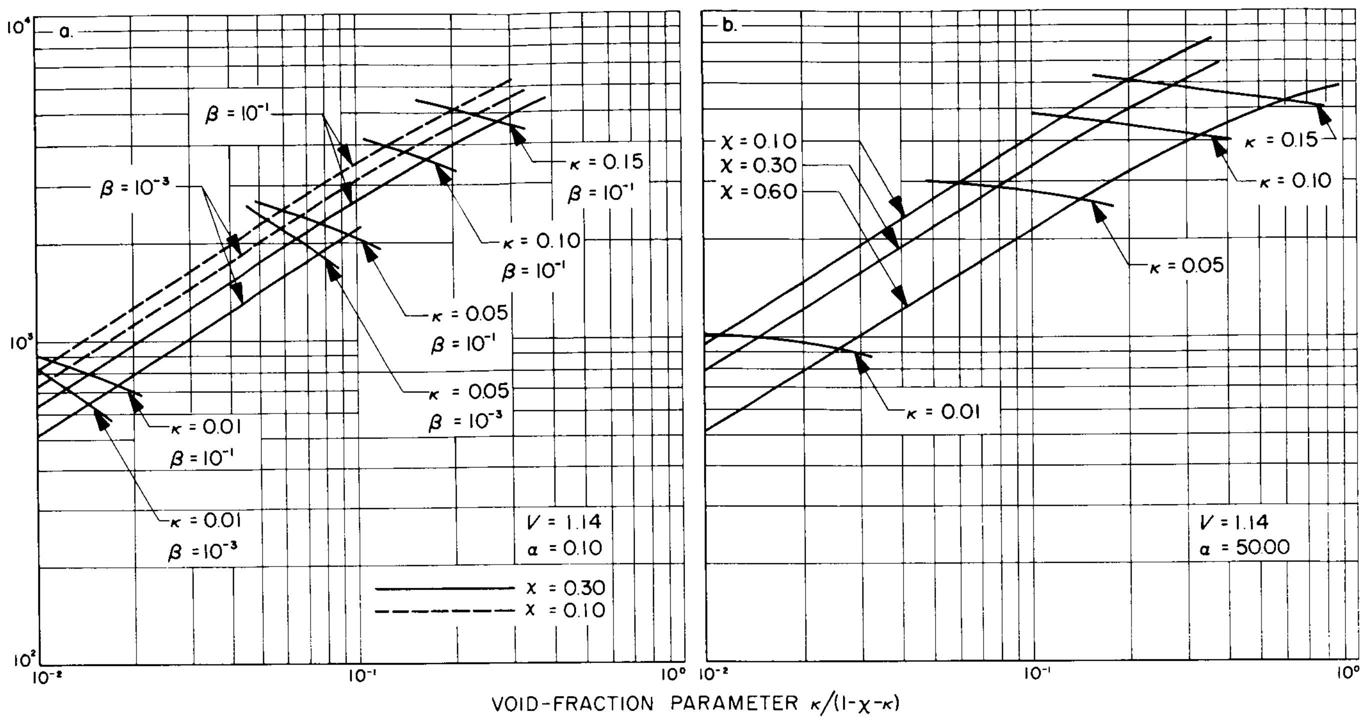

Fig. 8. Number of regenerative-cooling tubes as a function of void-fraction parameter, $V = 1.14$

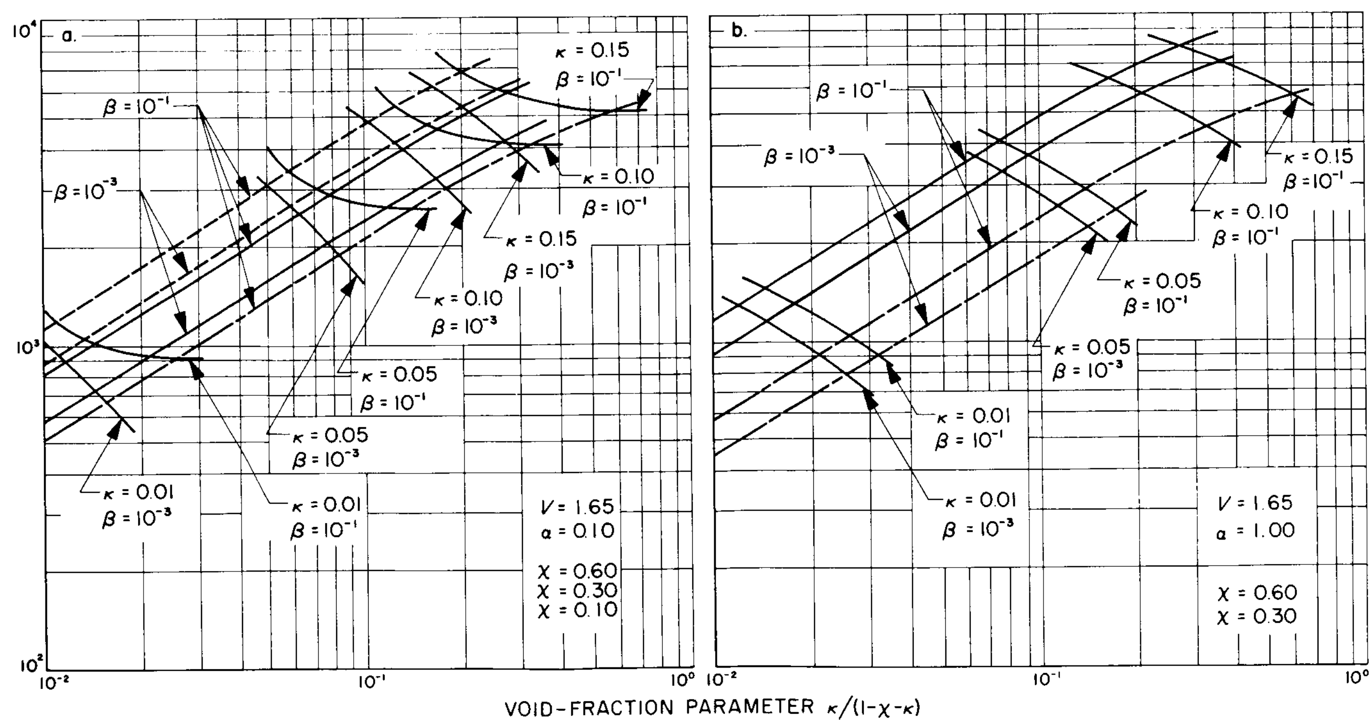

Fig. 9. Number of regenerative-cooling tubes as a function of void-fraction parameter, $V = 1.65$

IV. CONCLUSION

1. Within the limits of the present analysis, the quantity which effectively establishes the value of the vortex-tube parameters $\mathfrak{M}$, N_V, and r_V is the ratio of the radiation terms ε_c/β. To obtain vortex-tube flow rates compatible with an adequate separation process, $\varepsilon_c/\beta \sim 20$ for $\chi \sim 0.1$, and $\varepsilon_c/\beta \sim 5$ for $\chi \sim 0.6$. This, then, sets the range of the vortex-tube radius as 0.13 to 6.00 in. and the required number of tubes at 3000 to 20,000 for an Earth-satellite mission and 5000 to 100,000 for an Earth-escape mission. Whether or not these values of ε_c/β can be obtained while maintaining adequate system capabilities depends upon the solution of the thermal-radiation problem; that is, the determination of the value of ε_c.

2. The regenerative-cooling-tube parameters r_r and N_r are most sensitive to the cooling-tube void fraction κ. On the basis of heat transfer alone, small values of κ result in more efficient cooling. Other factors, however, such as pressure drops, must be considered in establishing proper values of κ.

NOMENCLATURE

A surface area, ft^2

a initial acceleration in units of gravitational constant

c exhaust velocity, ft/sec

F thrust, lb

f fraction of total fission power released in the solid region

g gravitation constant, ft/sec^2

h enthalpy, Btu/lb

h^* heat-transfer coefficient, $Btu/ft^2 - {}^\circ F\text{-hr}$

I specific impulse, sec

I^* ratio of specific impulses, $I^* = I_c/I_s$

k thermal conductivity, $Btu/ft - {}^\circ F\text{-hr}$

l length of cylindrical reactor, ft

$\dot{m}$ propellant mass flow, lb/sec

$\mathfrak{M}$ propellant flow rate per unit length of vortex tube, lb/ft-sec

N number of tubes

P power, Btu/sec

p power density, Btu/ft^3-sec

Pr Prandtl number

r tube radius, in.

s ratio of tank weight to propellant weight

T temperature, °R

V required velocity increment in units of exhaust velocity, corresponding to solid temperature

V^* volume, ft^3

v velocity of propellant in regenerative-cooling tubes, ft/sec

W weight, lb

α ratio of power generation per unit volume in solid region to that in cavity region

β thermal radiation parameter, $\beta = (\sigma \varepsilon_c A_c T_s)/(\dot{m} h_s)$

ε emissivity

ζ fraction of fission energy released in the cavity regions which appears as radiation and is attenuated in the solid region

θ thrust-to-weight ratio of an all-solid-fuel reactor with exhaust velocity c_s

κ volume fraction of regenerative tubes, $\kappa = V_r^*/V_R^*$

λ ratio of propellant weight to total vehicle weight

μ viscosity, lb/ft-sec

π ratio of payload weight to total weight

ρ specific weight, lb/ft^3

σ Stephan Boltzmann constant, $\sigma = 0.1737 \times 10^{-8}$ $Btu/(hr\ ft^2\ {}^\circ R^4)$

χ cavity void fraction, $\chi = V_c^*/V_R^*$

Subscripts

c cavity region

PL payload

R reactor

r regenerative-cooling tubes

S solid region

V vortex tubes

0 total

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